Applied Mathematics Letters 24 (2011) 687–691

Contents lists available at ScienceDirect

Applied Mathematics Letters

journal homepage: www.elsevier.com/locate/aml

Sharp bounds on the zeroth-order general Randić index of unicyclic graphs with given diameter ☆

Xiang-Feng Pan [a,*], Huiqing Liu [b], Meimei Liu [c]

[a] School of Mathematical Sciences, Anhui University, Hefei, Anhui, 230601, PR China

[b] School of Mathematics and Computer Science, Hubei University, Wuhan, Hubei, 430062, PR China

[c] Department of Statistics and Finance, University of Science and Technology of China, Hefei, Anhui, 230026, PR China

ARTICLE INFO

Article history:
Received 1 August 2010
Received in revised form 1 December 2010
Accepted 7 December 2010

Keywords:
Unicyclic graph
Zeroth-order general Randić index
Diameter
Sharp bound

ABSTRACT

Let G be a simple connected graph and α be a given real number. The zeroth-order general Randić index ${}^0R_\alpha(G)$ is defined as $\sum_{v\in V(G)}[d_G(v)]^\alpha$, where $d_G(v)$ denotes the degree of the vertex v of G. In this work, we give, for any $\alpha(\neq 0, 1)$, some sharp bounds on the zeroth-order general Randić index ${}^0R_\alpha$ of all unicyclic graphs with n vertices and diameter d.

1. Introduction

Given a simple connected graph $G = (V(G), E(G))$, where $V(G)$ and $E(G)$ denote the sets of vertices and edges of G, respectively, the Randić index of G is defined as [1]

$$R(G) = \sum_{(u,v)\in E(G)} [d_G(u)d_G(v)]^{-\frac{1}{2}},$$

where $d_G(u)$ denotes the degree of the vertex u of G. Randić himself demonstrated [1] that this index is well correlated with a variety of physico-chemical properties of various classes of organic compounds. The Randić index and some of its variants have received intensive attention and have been generalized in many ways. In particular, the zeroth-order general Randić index is defined in [2] as

$${}^0R_\alpha(G) = \sum_{v\in V(G)} [d_G(v)]^\alpha,$$

where α is a real number. Some nice results can be found in the literature [2–13].

In this work, we investigate extremal values of general ${}^0R_\alpha$ for simple connected unicyclic graphs of n vertices and diameter d and give some sharp bounds for this class of graphs. Since ${}^0R_0(G) = |V(G)|$ and ${}^0R_1(G) = 2|E(G)|$, we only consider $\alpha \neq 0, 1$.

☆ Partially supported by NNSFC (Nos. 10971114, 10901001 and 60772121), Innovation Group Fund of Hubei Provincial Department of Education (No. T200901), the young talents fund of universities of Anhui Province of China (No. 2010SQRL020), and Academic Innovation Team of Anhui University (KJTD001B).

* Corresponding author. Fax: +86 05513861737.
E-mail addresses: xfpan@ustc.edu (X.-F. Pan), hql_2008@163.com (H. Liu), a10614043@126.com (M. Liu).

0893-9659/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.aml.2010.12.006

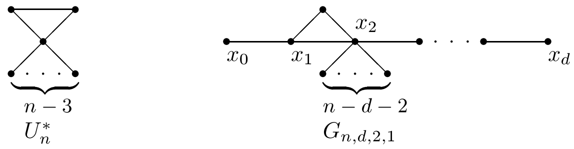

Fig. 1. The graphs in Introduction.

Now we introduce some definitions and notation. For undefined graph-theoretical terminology and notation the reader may refer to [14]. A *unicyclic* graph is a connected simple graph of n vertices and n edges. For two vertices $u, v \in V(G) (u \neq v)$, the *distance* between u and v, denoted by $d_G(u, v)$, is the number of edges in a shortest path joining u and v in G. The *diameter* of a graph G, denoted by $\mathrm{diam}(G)$, is the maximum distance between any two vertices of G. A *pendent vertex* of a graph is a vertex with degree 1. Let C_n and P_n denote the cycle and path of n vertices, respectively. An edge is said to be *subdivided* when it is deleted and replaced by a path of length 2 connecting its ends, the internal vertex of this path being a new vertex. A *subdivision* of a graph G is a graph that can be obtained from G by a sequence of edge subdivisions. Let $\mathscr{U}_{n,d}$ denote the set of all unicyclic graphs with order n and diameter d, where $1 \le d \le n-2$. Let U_n^* (shown in Fig. 1) denote the graph obtained from star S_n by adding a new edge between its two pendent vertices. Then $\mathscr{U}_{n,1} = \{C_3\}$ (if $n = 3$) or $\emptyset$ (if $n \ge 4$) and $\mathscr{U}_{n,2} = \{U_4^*, C_4\}$ (if $n = 4$) or $\{U_5^*, C_5\}$ (if $n = 5$) or $\{U_n^*\}$ (if $n \ge 6$). Denote by $D(G) = (d_1, d_2, \ldots, d_n)$ the degree sequence of the graph G, where d_i stands for the degree of the ith vertex of G and $d_1 \ge d_2 \ge \cdots \ge d_n$. Let $\mathscr{D}_n$ denote the set of sequences $D = (d_1, d_2, \ldots, d_n)$ of positive integers with $d_1 \ge d_2 \ge \cdots \ge d_n$ and $\sum_{i=1}^n d_i = 2n$. The *zeroth-order general Randić index* of a sequence $D = (d_1, d_2, \ldots, d_n)$ of positive integers is defined as

$$^0R_\alpha(D) = \sum_{i=1}^{n} d_i^\alpha,$$

where α is a real number. Note that $^0R_\alpha(G) = {}^0R_\alpha(D(G))$ for any graph G.

Let $D_1(n, d) = (n-d+1, 3, \overbrace{2, \ldots, 2}^{d-2}, \overbrace{1, \ldots, 1}^{n-d})$ and $f_1(n, d, \alpha) = {}^0R_\alpha(D_1(n, d))$.
Let

$$D_2(n,d) = \begin{cases} D(C_n) = (\overbrace{2, \ldots, 2}^{n}) & \text{if } d = \left\lfloor \dfrac{n}{2} \right\rfloor, \\ (3, \overbrace{2, \ldots, 2}^{n-2}, 1) & \text{if } \left\lfloor \dfrac{n}{2} \right\rfloor < d \le n-2, \end{cases}$$

and $f_2(n, d, \alpha) = {}^0R_\alpha(D_2(n, d))$, where $3 \le \lfloor \frac{n}{2} \rfloor \le d \le n-2$.

Let $P_d = x_0x_1 \ldots x_d$ be a path of length d. Let $G_{n,d,i,j}$ denote the graph obtained from P_d by attaching $n-d-1$ pendent vertices to x_i and joining one of the new pendent vertices to x_j. For example, $G_{n,d,2,1}$ is shown in Fig. 1. Let $\mathscr{G}_{n,d} = \{G_{n,d,i,j} | 2 \le i \le d-1 \text{ and } 1 \le j = i-1 \text{ or } i-2\}$.

2. Lemmas

In this section, we give some lemmas that will be used in the proof of our result.

Lemma 2.1 ([8]). *If $x - 2 \ge y \ge 1$, then*

$$\begin{cases} (x-1)^\alpha + (y+1)^\alpha < x^\alpha + y^\alpha & \text{if } \alpha < 0 \text{ or } \alpha > 1, \\ (x-1)^\alpha + (y+1)^\alpha > x^\alpha + y^\alpha & \text{if } 0 < \alpha < 1. \end{cases}$$

Lemma 2.2 ([8]). *If $x \ge y \ge 2$, then*

$$\begin{cases} (x+1)^\alpha + (y-1)^\alpha > x^\alpha + y^\alpha & \text{if } \alpha < 0 \text{ or } \alpha > 1, \\ (x+1)^\alpha + (y-1)^\alpha < x^\alpha + y^\alpha & \text{if } 0 < \alpha < 1. \end{cases}$$

Lemma 2.3. *If $D = (d_1, d_2, \ldots, d_n) \in \mathscr{D}_n$ and there are two integers d_i, d_j such that $d_i - 2 \ge d_j \ge 1$, then there exists another sequence of integers $D' \in \mathscr{D}_n$ satisfying*

$$\begin{cases} {}^0R_\alpha(D) > {}^0R_\alpha(D') & \text{for } \alpha < 0 \text{ or } \alpha > 1, \\ {}^0R_\alpha(D) < {}^0R_\alpha(D') & \text{for } 0 < \alpha < 1. \end{cases}$$

Proof. Let $d'_i = d_i - 1, d'_j = d_j + 1$ and $d'_k = d_k$ for each $k \in \{1, 2, \dots, n\} \setminus \{i, j\}$. Rearrange $d'_1, d'_2, \dots, d'_n$ as $d'_{i_1} \ge d'_{i_2} \ge \dots \ge d'_{i_n}$. It is not difficult to see that $D' = (d'_{i_1}, d'_{i_2}, \dots, d'_{i_n}) \in \mathscr{D}_n$ and, by Lemma 2.1,

$$^0R_\alpha(D) - {}^0R_\alpha(D') = d_i^\alpha + d_j^\alpha - (d_i - 1)^\alpha - (d_j + 1)^\alpha \begin{cases} >0 & \text{for } \alpha < 0 \text{ or } \alpha > 1, \\ <0 & \text{for } 0 < \alpha < 1. \end{cases}$$

Then the proof of this lemma is complete. □

Lemma 2.4. *If $D = (d_1, d_2, \dots, d_n) \in \mathscr{D}_n$ and there are two integers d_i, d_j such that $d_i \ge d_j \ge 2$, then there exists another sequence of integers $D' \in \mathscr{D}_n$ satisfying*

$$\begin{cases} ^0R_\alpha(D) < {}^0R_\alpha(D') & \text{for } \alpha < 0 \text{ or } \alpha > 1, \\ ^0R_\alpha(D) > {}^0R_\alpha(D') & \text{for } 0 < \alpha < 1. \end{cases}$$

Proof. Let $d'_i = d_i + 1, d'_j = d_j - 1$ and $d'_k = d_k$ for each $k \in \{1, 2, \dots, n\} \setminus \{i, j\}$. Rearrange $d'_1, d'_2, \dots, d'_n$ as $d'_{i_1} \ge d'_{i_2} \ge \dots \ge d'_{i_n}$. It is not difficult to see that $D' = (d'_{i_1}, d'_{i_2}, \dots, d'_{i_n}) \in \mathscr{D}_n$ and, by Lemma 2.2,

$$^0R_\alpha(D) - {}^0R_\alpha(D') = d_i^\alpha + d_j^\alpha - (d_i + 1)^\alpha - (d_j - 1)^\alpha \begin{cases} <0 & \text{for } \alpha < 0 \text{ or } \alpha > 1, \\ >0 & \text{for } 0 < \alpha < 1. \end{cases}$$

Then the proof of this lemma is complete. □

The proof of the following lemma is not difficult and thus is omitted here (for a reference, see the Appendix).

Lemma 2.5. *Let $G \in \mathscr{U}_{n,d}$, where $3 \le d \le n - 2$. Then $D(G) = D_1(n, d)$ if and only if $G \in \mathscr{G}_{n,d}$.*

3. Results

Theorem 3.1. *Let $G \in \mathscr{U}_{n,d}$, where $3 \le d \le n - 2$. Then*

$$\begin{cases} ^0R_\alpha(G) \le f_1(n, d, \alpha) & \text{for } \alpha < 0 \text{ or } \alpha > 1, \\ ^0R_\alpha(G) \ge f_1(n, d, \alpha) & \text{for } 0 < \alpha < 1. \end{cases}$$

Furthermore, the equalities hold if and only if $G \in \mathscr{G}_{n,d}$.

Proof. We only show Theorem 3.1 for the case of $\alpha < 0$ or $\alpha > 1$, while the proof of Theorem 3.1 for $0 < \alpha < 1$ is similar and thus omitted.

Assume $\alpha < 0$ or $\alpha > 1$ in the following.

Note that if $G \in \mathscr{G}_{n,d}$, then, by Lemma 2.5, $D(G) = D_1(n, d)$ and $^0R_\alpha(G) = f_1(n, d, \alpha)$.

Assume $G \in \mathscr{U}_{n,d}$ with $D(G) = (d_1, d_2, \dots, d_{n-z}, \overbrace{1, 1, \dots, 1}^{z})$, where $d_{n-z} \ge 2$.

Claim A. *$0 \le z \le n - d$.*

In fact, let $P = x_0x_1 \dots x_d$ and C be the shortest path of length d and the cycle in G, respectively; then $d_G(x_i) \ge 2$ for all $i = 1, 2, \dots, d - 1$ and there exists at least one vertex in $V(C) \setminus V(P)$. Thus there are at least d vertices of degree greater than 1. Then $z \le n - d$. So Claim A is true.

Let $D^* = (n + 2 - d, \overbrace{2, \dots, 2}^{d-1}, \overbrace{1, \dots, 1}^{n-d})$. Then $D^* \in \mathscr{D}_n$ and, by the proof of Lemma 2.4, $^0R_\alpha(G) \le {}^0R_\alpha(D^*)$. In order for the equality to hold, we must have $D(G) = D^*$.

Claim B. *$D(G) = D^*$ if and only if G is isomorphic to a subdivision of U^*_{n+3-d}, denoted by $G_1(n)$, obtained from U^*_{n+3-d} by $d - 3$ successive edge subdivisions.*

Obviously, if $G \cong G_1(n)$, then $D(G) = D^*$. Conversely, assume $D(G) = D^*$. Let $v \in V(G)$ be the vertex of maximum degree $n + 2 - d$. Then v is on the unique cycle of G (otherwise, there must be another vertex of degree more than 2). Thus G is isomorphic to some $G_1(n)$. So Claim B is true.

It is easy to see that $\mathrm{diam}(G_1(n)) \le d - 1$. Thus $G_1(n) \notin \mathscr{U}_{n,d}$. From the proof of Lemma 2.4, for each $D(G)$, where $G \in \mathscr{U}_{n,d}$, there is a sequence of integer sequences $D'_1, D'_2, \dots, D'_s$ in $\mathscr{D}_n$ such that $D'_1 = D(G), D'_s = D^*$ and D'_{k+1}, where $k = 1, 2, \dots, s - 1$, is obtained from D'_k by an operation similar to that from D to D' in the proof of Lemma 2.4.

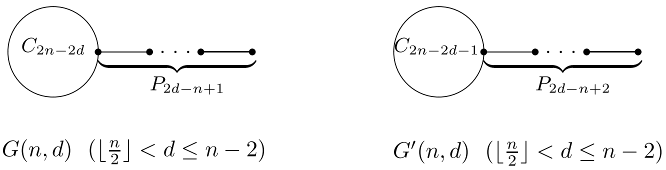

Fig. 2. The graphs in Theorem 3.2.

Then ${}^0R_\alpha(D_1') < {}^0R_\alpha(D_2') < \cdots < {}^0R_\alpha(D_s')$. For all $G \in \mathscr{U}_{n,d}$, either $D_{s-1}' = (n+1-d, \overbrace{2, \ldots, 2}^{d}, \overbrace{1, \ldots, 1}^{n-d-1})$ or $D_{s-1}' = (n+1-d, 3, \overbrace{2, \ldots, 2}^{d-2}, \overbrace{1, \ldots, 1}^{n-d}) = D_1(n,d)$. By Lemma 2.1,

$$ {}^0R_\alpha((n+1-d, \overbrace{2, \ldots, 2}^{d}, \overbrace{1, \ldots, 1}^{n-d-1})) - {}^0R_\alpha(D_1(n,d)) = 2 \cdot 2^\alpha - 3^\alpha - 1^\alpha < 0. $$

Then ${}^0R_\alpha((n+1-d, \overbrace{2, \ldots, 2}^{d}, \overbrace{1, \ldots, 1}^{n-d-1})) < {}^0R_\alpha(D_1(n,d))$.

Therefore ${}^0R_\alpha(G) \leq {}^0R_\alpha(D_1(n,d)) = f_1(n,d,\alpha)$ for all $G \in \mathscr{U}_{n,d}$. In order to have ${}^0R_\alpha(G) = f_1(n,d,\alpha)$, we must have $D(G) = D_1(n,d)$. By Lemma 2.5, $G \in \mathscr{G}_{n,d}$.

So Theorem 3.1 for the case of $\alpha < 0$ or $\alpha > 1$ is true. □

Theorem 3.2. *Let $G \in \mathscr{U}_{n,d}$, where $3 \leq \lfloor \frac{n}{2} \rfloor \leq d \leq n-2$. Then*

$$ \begin{cases} {}^0R_\alpha(G) \geq f_2(n,d,\alpha) & \text{for } \alpha < 0 \text{ or } \alpha > 1, \\ {}^0R_\alpha(G) \leq f_2(n,d,\alpha) & \text{for } 0 < \alpha < 1. \end{cases} $$

Furthermore, the equalities hold if and only if $G \cong C_n$ for $d = \lfloor \frac{n}{2} \rfloor$ or $G \cong G_3(n,d)$ or $G_3'(n,d)$ (shown in Fig. 2) for $3 \leq \lfloor \frac{n}{2} \rfloor < d \leq n-2$.

Proof. We only show Theorem 3.2 for the case of $\alpha < 0$ or $\alpha > 1$, while the proof of Theorem 3.2 for $0 < \alpha < 1$ is similar and thus omitted.

Assume $\alpha < 0$ or $\alpha > 1$ in the following.

Firstly, it is easy to check that if $G \cong C_n$ or $G(n,d)$ or $G'(n,d)$, then $G \in \mathscr{U}_{n,d}$ and $D(G) = D_2(n,d)$. Thus ${}^0R_\alpha(G) = f_2(n,d,\alpha)$.

Let $G \in \mathscr{U}_{n,d}$. Then, by Lemma 2.3, ${}^0R_\alpha(G) \geq {}^0R_\alpha\left(D_2\left(n, \lfloor \frac{n}{2} \rfloor\right)\right)$, where $D_2\left(n, \lfloor \frac{n}{2} \rfloor\right) = (\overbrace{2, \ldots, 2}^{n})$. In order for the equality to hold, we must have $D(G) = D_2\left(n, \lfloor \frac{n}{2} \rfloor\right)$. It is not difficult to see that $D(G) = D_2\left(n, \lfloor \frac{n}{2} \rfloor\right)$ if and only if $G \cong C_n$.

Note that $\mathrm{diam}(C_n) = \lfloor \frac{n}{2} \rfloor$. If $3 \leq \lfloor \frac{n}{2} \rfloor < d \leq n-2$, then $C_n \notin \mathscr{U}_{n,d}$. From the proof of Lemma 2.3, for each $D(G)$, where $G \in \mathscr{U}_{n,d}$, there is a sequence of integer sequences $D_1', D_2', \ldots, D_s'$ in $\mathscr{D}_n$ such that $D_1' = D(G)$, $D_s' = D_2\left(n, \lfloor \frac{n}{2} \rfloor\right)$ and D_{k+1}', where $k = 1, 2, \ldots, s-1$, is obtained from D_k' by an operation similar to that going from D to D' in the proof of Lemma 2.3.

Then ${}^0R_\alpha(D_1') > {}^0R_\alpha(D_2') > \cdots > {}^0R_\alpha(D_s')$ and for all $G \in \mathscr{U}_{n,d}$, $D_{s-1}' = (3, \overbrace{2, \ldots, 2}^{n-2}, 1) = D_2(n,d)$.

Therefore ${}^0R_\alpha(G) \leq {}^0R_\alpha(D_2(n,d)) = f_2(n,d,\alpha)$ for all $G \in \mathscr{U}_{n,d}$. In order to have ${}^0R_\alpha(G) = f_2(n,d,\alpha)$, we must have $D(G) = D_2(n,d)$ and thus $G \cong G(n,d)$ or $G'(n,d)$ where $3 \leq \lfloor \frac{n}{2} \rfloor < d \leq n-2$.

So Theorem 3.2 for the case of $\alpha < 0$ or $\alpha > 1$ is true. □

4. Remark

In this work, we give partial sharp bounds for the zeroth-order general Randić index for all unicyclic graphs with given diameter. For $3 \leq d < \lfloor \frac{n}{2} \rfloor$, the upper (resp. lower) bounds of ${}^0R_\alpha(G)$ for $0 < \alpha < 1$ (resp. $\alpha < 0$ or $\alpha > 1$) for $G \in \mathscr{U}_{n,d}$ are still not established.

Acknowledgements

The authors would like to express their sincere gratitude to the referees for their valuable suggestions, which led to great deal of improvement in this work.

Appendix

Proof of Lemma 2.5. Firstly, if $G \in \mathscr{G}_{n,d}$, then it is easy to see that $G \in \mathscr{U}_{n,d}$ and $D(G) = D_1(n, d)$.

Secondly, let $G \in \mathscr{U}_{n,d}$ with $D(G) = D_1(n, d)$, where $3 \leq d \leq n-2$. Let $P = x_0x_1 \ldots x_d$ be a shortest path of length d in G. Then $d_G(x_i) \geq 2$ for all $i = 1, \ldots, d-1$. Since $D(G) = D_1(n, d)$, there are just d vertices of degree greater than 1 in G. If one of the vertices with maximum degree is not in $\{x_1, x_2, \ldots, x_{d-1}\}$, then there exists $i \in \{1, 2, \ldots, d-1\}$ such that $d_G(x_i) = 3$ and $d_G(x_j) = 2$ for all $j \in \{1, 2, \ldots, d-1\} \setminus \{i\}$, since G has only two vertices of degree ≥ 3. Evidently, $d_G(x_k) \neq n-d+1$ for $k = 0, d$ (otherwise, the diameter of G is greater than d). Then $V(C) \cap V(P) = \emptyset$, where C is the unique cycle of G. Thus there are at least $d+2$ vertices with degree ≥ 2. This is a contradiction to $D(G) = D_1(G)$. Consequently, there exists $i' \in \{1, 2, \ldots, d-1\}$ such that $d_G(x_{i'}) = n-d+1$. If $d_G(x_{i'}) = n-d+1 = 3$, then, by the previous reasoning, there exists $j' \in \{1, 2, \ldots, d-1\} \setminus \{i'\}$ with $d_G(x_{j'}) = 3$, since there are two vertices of maximum degree 3 in G. Then $|j'-i'| \leq 2$ and there is a vertex u of G such that $ux_{i'}, ux_{j'} \in E(G)$, since $G \in \mathscr{U}_{n,d}$ and $D(G) = D_1(n, d)$. Without loss of generality, assume $j' < i'$. Then $G \cong G_{n,n-2,i',j'}$, where $1 \leq j' = i'-1$ or $i'-2$. Thus $G \in \mathscr{G}_{n,n-2}$. Otherwise, if $d_G(x_{i'}) = n-d+1 \geq 4$, then we claim that the unique vertex v of degree 3 is in $\{x_1, x_2, \ldots, x_{d-1}\} \setminus \{x_{i'}\}$. In fact, if $v \in V(G) \setminus V(P)$, then there are at least $d+1$ vertices with degree ≥ 2, a contradiction to $D(G) = D_1(n, d)$. So $v \in V(P)$. If $v \in \{x_0, x_d\}$, then G has more than $d-2$ vertices of degree 2, a contradiction to $D(G) = D_1(n, d)$. Then $v \in \{x_1, x_2, \ldots, x_{d-1}\} \setminus \{x_{i'}\}$. So the claim is true. Assume $v = x_{j'}$, where $j' \in \{1, 2, \ldots, d-1\} \setminus \{i'\}$. Then $|j'-i'| \leq 2$. Without loss of generality, assume $j' < i'$ (otherwise, we can relabel the vertices of P as $x_0, x_1, \ldots, x_d$ one by one from the opposite direction). Then $G \cong G_{n,d,i',j'}$, where $1 \leq j' = i'-1$ or $i'-2$. Thus $G \in \mathscr{G}_{n,d}$. □

References

[1] M. Randić, On the characterization of molecular branching, J. Am. Chem. Soc. 97 (1975) 6609–6615.
[2] X. Li, J. Zheng, A unified approach to the extremal trees for different indices, MATCH Commun. Math. Comput. Chem. 54 (2005) 195–208.
[3] S. Chen, H. Deng, Extremal $(n, n+1)$-graphs with respected to zeroth-order general Randić index, J. Math. Chem. 42 (2007) 555–564.
[4] Y. Hu, X. Li, Y. Shi, T. Xu, Connected (n, m)-graphs with minimum and maximum zeroth-order general Randić index, Discrete Appl. Math. 155 (2007) 1044–1054.
[5] Y. Hu, X. Li, Y. Shi, T. Xu, I. Gutman, On molecular graphs with smallest and greatest zeroth-order general Randić index, MATCH Commun. Math. Comput. Chem. 54 (2005) 425–434.
[6] H. Hua, H. Deng, On unicycle graphs with maximum and minimum zeroth-order general Randić index, J. Math. Chem. 41 (2007) 173–181.
[7] X. Li, H. Zhao, Trees with the first three smallest and largest generalized topological indices, MATCH Commun. Math. Comput. Chem. 50 (2004) 57–62.
[8] A. Lin, R. Luo, X. Zha, On sharp bounds of the zero-order Randić index of certain unicyclic graphs, Appl. Math. Lett. 22 (2009) 585–589.
[9] X.-F. Pan, N.-N. Lv, Sharp bounds of the zeroth-order general Randić index of bicyclic graphs with given pendent vertices, Discrete Appl. Math. (2010) doi:10.1016/j.dam.2010.10.018.
[10] L. Pavlović, Maximal value of the zeroth-order Randić index, Discrete Appl. Math. 127 (2003) 615–626.
[11] L. Pavlović, M. Lazić, T. Aleksić, More on Connected (n, m)-graphs with minimum and maximum zeroth-order general Randić index, Discrete Appl. Math. 157 (2009) 2938–2944.
[12] H. Wang, H. Deng, Unicycle graphs with maximum generalized topological indices, J. Math. Chem. 42 (2007) 119–124.
[13] B. Zhang, B. Zhou, On zeroth-order general Randić indices of trees and unicyclic graphs, MATCH Commun. Math. Comput. Chem. 58 (2007) 139–146.
[14] J.A. Bondy, U.S.R. Murty, Graph Theory, Springer, Berlin, 2008.